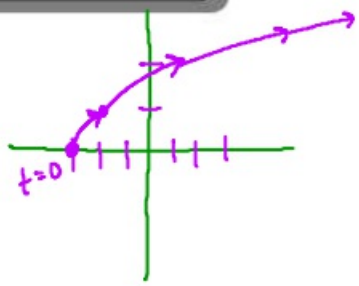
What you'll Learn About
How to take the derivative of functions in Parametric Form

Graph the parametric function given

A) $x = t^2 - 3 \quad y = t \quad t \geq 0$

t	x	y
0	-3	0
1	-2	1
2	1	2



B) Find the derivative of the function at t=5

$\frac{dy}{dx}$

$x = t^2 - 3 \quad y = t$

$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 1$

$\left.\frac{dy}{dx}\right|_{t=5} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{1}{2t} = \frac{1}{10}$

C) Find the equation of the tangent line at t=1

$x = 3t \quad y = 9t^2 \quad t = 1 \quad (3, 9)$

$y = 9 + 6(x-3)$

$\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = 18t \quad \left.\frac{dy}{dx}\right|_{t=1} = \frac{18t}{3} = 6t = 6$

D) Find the equation of the tangent line at $\theta = \frac{\pi}{4}$

$x = \cos\theta \quad y = \sin\theta \quad \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$y = \frac{\sqrt{2}}{2} - 1\left(x - \frac{\sqrt{2}}{2}\right)$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{\cos\theta}{-\sin\theta} = \frac{\cos \pi/4}{-\sin \pi/4} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

$x = \left[\sec(2\pi)\right]^2 - 1$

$x = 1^2 - 1 = 0$

$y = \tan[2\pi]$

E) Find the equation of the tangent line at $t = \pi$ (0, 0)

$x = \sec^2(2t) - 1 \quad y = \tan(2t)$

Vertical Line

$x = 0$

$x = \left[\sec(2t)\right]^2 - 1$

$y = \frac{\sin[2\pi]}{\cos[2\pi]} = \frac{0}{1} = 0$

$\frac{dx}{dt} = 2\left[\sec(2t)\right] \cdot \left[\sec(2t)\tan(2t) \cdot 2\right]$

$\frac{dy}{dt} = \sec^2(2t) \cdot 2$

$\frac{dy}{dx} = \frac{2\sec^2(2t)}{4\sec^2(2t)\tan(2t)}$

$\frac{dy}{dx} = \frac{1}{2\tan(2t)}$

$\frac{dy}{dx} = \frac{1}{2\tan(2\pi)} = \frac{1}{0} \rightarrow$ UNDEFINED

$y = 64 + 12(x-1)$

A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Determine the equation of the line tangent to the graph of C at the point (1, 64)?

$\frac{dy}{dx} = \frac{3t^2}{2t-4}$

$\left.\frac{dy}{dx}\right|_{t=4} = \frac{48}{4} = 12$

x y

$1 = t^2 - 4t + 1$

$0 = t^2 - 4t$

$0 = t(t-4)$

$t = 0 \quad t = 4$

$64 = t^3$

$4 = t$

Determine the horizontal and vertical tangents for the parametric curve

A) $x = 1 - t \quad y = t^2 - 4t$

Horizontal Tangents

$\frac{dy}{dx} = 0$ $\left(\frac{dy}{dt} = 0\right)$

Vertical Tangent

$\frac{dy}{dx} = \text{und}$ $\left(\frac{dx}{dt} = 0\right)$

H.T

$2t - 4 = 0$

$t = 2$

$(-1, -4)$

V.T

$-1 \neq 0$

B) $x = \cos\theta \quad y = 2\sin(2\theta)$ $\quad 0 \le \theta \le 2\pi$

Vertical Tangent

$-\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$

Horizontal Tangent

$2\cos(2\theta) \cdot 2 = 0$

$4\cos 2\theta = 0$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$